Universal Power-law Decay in Hamiltonian Systems?

The understanding of the asymptotic decay of correlations and of the distribution of Poincaré recurrence times $P(t)$ has been a major challenge in the field of Hamiltonian chaos for more than two decades. In a recent Letter, Chirikov and Shepelyansky [1] claimed the universal decay $P(t) \sim t^{-3}$ for Hamiltonian systems. Their reasoning is based on renormalization arguments and numerical findings for the sticking of chaotic trajectories near a critical golden torus in the standard map. We performed extensive numerics and find clear deviations from the predicted asymptotic exponent of the decay of $P(t)$. We thereby demonstrate that even in the supposedly simple case, when a critical golden torus is present, the fundamental question of asymptotic statistics in Hamiltonian systems remains unsolved.

As in Ref. [1] we study the standard map

$$q_{n+1} = q_n + p_n \bmod 2\pi \qquad p_{n+1} = p_n + K \sin q_{n+1} \ , \quad (1)$$

at $K = K_c = 0.97163540631$, where the golden torus is critical (Fig. 1, inset). We determine the Poincaré recurrence time distributions $P(t)$ for trajectories starting below and above the critical golden torus by using the same numerical approach as in Ref. [1]. By considerably increasing the statistics we are able to extend the distribution by almost two orders of magnitude in recurrence times. We verify that our statistical data are not affected by the unavoidable finite numerical precision by comparing data for double (≈16 significant digits) and quadruple (≈32 digits) precision. The data for approaching the critical golden torus from above and below are presented in Fig. 1. For times $t < 10^8$ our data agree with the results presented in Fig. 2 of Ref. [1]. For larger times, however, we find strong deviations from the predicted universal power law $P(t) \sim t^{-3}$ (dashed lines in Fig. 1). The deviations might be explained in two ways: The onset of the claimed asymptotic decay might occur for larger times, which is in contradiction to the prefactors determined in Ref. [1]. On the other hand, the long-time trapping of chaotic trajectories might be dominated by islands of stability (non-principal resonances) that are neglected by the renormalization arguments. In fact, the latter possibility is supported by a detailed investigation [2].

If even in the supposedly simple case of a critical golden torus the decay $P(t) \sim t^{-3}$ is not observed, the claim for a universal existence of this decay cannot be maintained. It thus remains a fundamental challenge in the field of Hamiltonian chaos whether the asymptotic behavior of $P(t)$ follows a universal power law and what the value of its exponent would be.

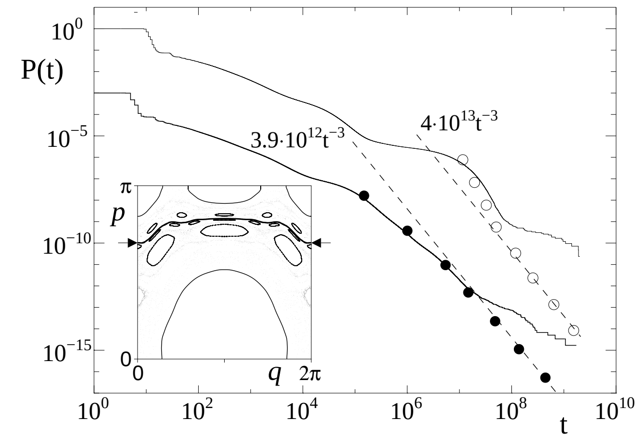

FIG. 1. The Poincaré recurrence time distribution $P(t)$ for the standard map at $K = K_c$ for trajectories approaching the critical golden torus from above (upper curve) and from below (lower curve, shifted by 10^{-3}). For large times we find clear deviations from the predictions of Ref. [1] (dashed lines and symbols). Inset: Phase space of the symmetrized standard map at $K = K_c$, with arrows pointing at the critical golden torus.

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